Geometry of level sets of random fields

Kac-Rice formulas, Hermite expansions and applications

Abridged preprint sample (about 10% of the book)

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Published version. This is an abridged preprint sample. The full book is published by Springer:
SpringerLink: https://link.springer.com/book/9783032291431
DOI: 10.1007/978-3-032-29143-1

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About this abridged sample

This file is a short sample of the book *Geometry of level sets of random fields: Kac-Rice formulas, Hermite expansions and applications*. It is not the full book. All proofs are removed. We keep the statements of the main results and we summarise each chapter in plain words. The full bibliography of the book is kept at the end, so the reader can find every reference.

The full book is published by Springer. The published version should be used for any precise statement, hypothesis or proof. Links to the published book are given on the title page above.

Introduction

The study of level sets started with the works of Kac (1943) and Rice (1945), who looked at the number of real roots of a random Gaussian polynomial or process. Itô (1963) gave the first general and rigorous proof of what is now called the Kac-Rice formula. Kac studied the number of real roots of a random Gaussian polynomial of degree d and proved that the mean number N_d behaves like

$$\mathbb{E}(N_d) \simeq \frac{2}{\pi} \log(d).$$

So a random polynomial has few real roots. The next question is the size of the fluctuations around this mean. This was answered by Maslova (1974), who computed the variance, and Maslova (1975), who proved a central limit theorem using the dependence approximation of Malevich (1969), today called the "Malevich method" (see also Cuzick (1976)).

At the start, the focus was the number of roots, or more generally the number of crossings of a level u, for a process $\mathbb{R} \to \mathbb{R}$, mostly Gaussian. The reference book on this is Cramér and Leadbetter (1967). The proof of the Kac-Rice formula is in essence a change of variables. Federer (1969) introduced the area and co-area formulas, which are strong tools to give general and short proofs of the Kac-Rice formula.

After the paper Adler and Hasofer (1976) and the book Adler (1981), the field moved to random fields, starting with the Euler characteristic of the excursion set above a level u. The perimeter or surface of the level set was first studied by Switzer (1976) under isotropy, and later under weaker conditions in Wschebor (1982), Wschebor (1985) and Benzaquen and Cabaña (1982). When the dimension D of the parameter space is larger than the dimension d of the image space, the level set $\mathcal{L}_u$ is in general a set of dimension $D - d$, and the Kac-Rice formula gives the mean of a measure instead of a number of roots. Some fields are even defined on a manifold, as in Azaïs and Wschebor (2004) and Auffinger et al. (2013).

The classical way to study the measure of nodal and level sets is to look, in this order, at the expectation, the variance and the central limit theorem (CLT). After the work of Maslova, new methods appeared. Nualart and Peccati (2005) proposed a method for Gaussian spaces, the fourth moment theorem, linked to the Hermite decomposition of random variables. The Hermite decomposition of the measure of the level set started with Slud (1991a) and Kratz and León (2001); the fourth moment theorem was used for level sets in Estrade and León (2016) and Azaïs et al. (2016). This is now a standard method to prove a CLT for the number of roots or for the measure of level sets. Through Arcones' inequality, the Hermite method also gives bounds on moments that the Kac-Rice formula cannot reach, for example the positivity of some limit variances. The two methods complete each other.

At the beginning of the 21$^{\text{st}}$ century several new models were studied: non-Gaussian random polynomials, random series of functions, series of spherical harmonics, and shot noises. Most of these are not Gaussian. The classical books on the Kac-Rice formula, namely Cramér and Leadbetter (1967), Adler (1981), Adler and Taylor (2009), Azaïs and Wschebor (2009) and Berzin et al. (2022), do not cover these models. There was a need for a new approach that:

- gives a new proof of the Kac-Rice formula that also works in the non-Gaussian case, and that holds under weak conditions even in the Gaussian case;
- gives a simple presentation of the minimal notions of geometry;

- covers the Hermite expansion and the fourth moment theorem;
- shows recent applications in Physics, high-dimensional statistics, kernel regression and critical points.

The first four chapters give the methods. Chapter 1 treats the number of roots of a process $\mathbb{R} \to \mathbb{R}$, with counterexamples that show why a Kac-Rice formula valid only for almost every level (as in Zähle (1984) and Brillinger (1972)) is not enough. Chapter 2 treats fields from $\mathbb{R}^D$ to $\mathbb{R}^d$ with $D \geq d$; it proves the Kac-Rice formula for every level, in the Gaussian and the non-Gaussian case, with examples. Chapter 3 collects the explicit Gaussian computations. Chapter 4 gives a simple presentation of Hermite expansions, the diagram formula, Arcones' inequality and the fourth moment theorem, and ends with a CLT for the measure of the level set. The next chapters give applications: critical points (Chapter 5), high-dimensional statistics and the spacing test (Chapter 6), shot noise (Chapter 7) and a review of related works (Chapter 8).

All the authors were influenced, directly or not, by the work of *Mario Wschebor*. This book continues his work and is dedicated to his memory.

Chapter 1

Kac-Rice formula, dimension 1

Summary

This chapter treats one-dimensional Kac-Rice formulas. They count the level crossings of a random process $X : \mathbb{R} \to \mathbb{R}$, that is the number N_u of points t where $X(t) = u$. The chapter separates with care three forms of the formula:

- a formula valid for *almost every* level u;
- an *upper bound* formula, valid for every level;
- the *true* Kac-Rice formula, valid for *every* level, hence also for a fixed level such as $u = 0$.

The almost-every-level formula is easy to obtain but often not enough, because applications need a fixed level, very often $u = 0$. Two counterexamples (a random line and a "poor" shot noise) show that the formula can fail at a particular level. The reason is the presence of a local extremum exactly at the level u. The tool that excludes this bad event is a Bulinskaya-type lemma: with probability one there is no point where $X(t) = u$ and $X'(t) = 0$. For Gaussian processes this holds automatically.

The chapter gives a direct (ad hoc) proof in the Gaussian case and postpones the non-Gaussian case to the next chapter. It then treats higher factorial moments of N_u, and ends with recent results on the finiteness of these moments, in the Gaussian and the non-Gaussian case.

Main results

For a process with smooth enough paths, the Kac-Rice formula for almost every level u reads

$$\mathbb{E}\big(N_u(X, B)\big) = \int_B \mathbb{E}\big(\, |X'(t)| \mid X(t) = u \,\big)\, p_{X(t)}(u)\, \mathrm{d}t.$$

The central result of the chapter is that, for a Gaussian process with C^1 mean, C^2 covariance and positive variance, the same formula holds for *every* level u and every Borel set B. For a stationary standardised Gaussian process this gives the classical closed form

$$\mathbb{E}\big(N_u\big) = |B|\, \sqrt{\lambda_2}\, \frac{e^{-u^2/2}}{\pi},$$

where λ_2 is the second spectral moment. The Gaussian Kac-Rice formula also holds at order k for the factorial moments $\mathbb{E}\big(N_u(N_u - 1) \cdots (N_u - k + 1)\big)$, written as a k-fold integral of a conditional expectation.

The last part of the chapter studies when these moments are finite. For a stationary Gaussian process, Geman's condition gives a necessary and sufficient condition for the second moment of N_u to be finite: it asks that an integral built from $\sigma^2(\tau) = \mathrm{Var}\big(X'(0) \mid X(0), X(\tau)\big)$ converges near $\tau = 0$. General methods are given for higher moments and for the non-Gaussian case.

Chapter 2

Multidimensional methods

Summary

This is the central chapter. It develops the Kac-Rice formula for a random field $X : T \subset \mathbb{R}^D \to \mathbb{R}^d$ with $D \geq d$. When $D = d$ the level set is, in general, a set of isolated points and the formula counts them. When $D > d$ the level set has dimension $D - d$ and the formula gives the mean of its $(D - d)$-dimensional Hausdorff measure.

The chapter first recalls the minimal geometry: the area, co-area and Crofton formulas. A main contribution is a new and stronger Bulinskaya-type lemma. It says that the bad part of the level set, where the Jacobian loses rank, has zero $(D - d)$-measure. The proof needs only weak conditions: C^1 paths and bounded densities. It works for Gaussian and non-Gaussian fields.

With this lemma the chapter proves the Kac-Rice formula for every level, again for Gaussian and non-Gaussian fields. The proof first treats $D = d$, and then extends to $D > d$ with the Crofton formula. Extensions are given for fields on manifolds, for higher moments and for weighted integrals over the level set. Many examples are treated: functions of Gaussian fields, sums of parametrised functions, the mean number of critical points of a likelihood, gravitational microlensing and shot noise. The chapter ends with recent results on the finiteness of the second moment and of higher moments of the measure of the level set.

Main results

The Bulinskaya-type lemma holds under the single integrability assumption that $\int_T p_{X(t)}(v)\,\mathrm{d}t$ is bounded for v near u. Then, almost surely, the set of points with $X(t) = u$ and degenerate Jacobian has zero $(D - d)$-measure. Under C^1 paths, bounded continuous density of $X(t)$ and continuity of the conditional laws, the Kac-Rice formula for every level u and every Borel set B takes the form

$$\mathbb{E}\big(\sigma_{D-d}(\mathcal{L}_u(B))\big) = \int_B \mathbb{E}\big(\,\Delta(t) \mid X(t) = u\,\big)\, p_{X(t)}(u)\,\mathrm{d}t,$$

where $\Delta(t)$ is the Jacobian factor (the number of roots when $D = d$, a Hausdorff density factor when $D > d$). A higher-moment version gives the k-th factorial moment as a k-fold integral. For the finiteness of moments: when $d < D$ the second moment of the measure of the level set is always finite on compact sets; when $D = d$ the second moment is finite under an integrability condition on the derivatives of the covariance, with a necessary and sufficient condition in the stationary case.

Chapter 3

Distributions, expectations and moments for Gaussian processes and fields

Summary

This chapter is a collection of the explicit Gaussian computations needed to apply the Kac-Rice formula. It is not meant to be read in a linear way; the reader should go to the section for the problem at hand. It treats four cases: stationary Gaussian processes ($\mathbb{R} \to \mathbb{R}$), non-stationary Gaussian processes, stationary Gaussian fields, and isotropic Gaussian fields, both in flat space and on the sphere.

For stationary processes the chapter recalls the differentiability condition: the process is ℓ times differentiable in quadratic mean if and only if the spectral moment $\lambda_{2\ell}$ is finite. It then gives the joint laws of the process and its derivatives, the mean number of crossings and of critical points, and detailed second-moment expressions, with their behaviour as two points come close.

For fields the chapter starts with Bochner's theorem, which describes valid covariances through a spectral measure. It gives special attention to Schoenberg covariances, valid in every dimension, seen as Laplace transforms of a measure. For isotropic fields the Hessian follows a scaled Gaussian Orthogonal Ensemble (GOE) law, which is the base for the critical-point computations of later chapters. The spherical case is treated in detail; this computation does not appear elsewhere.

Main results

For a stationary standardised Gaussian process the mean number of u-crossings on B is

$$\mathbb{E}\big(N_u(B)\big) = \frac{|B|}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}\,\exp\Big(-\frac{u^2}{2\lambda_0}\Big),$$

and the mean number of critical points is $\frac{|B|}{\pi}\sqrt{\lambda_4/\lambda_2}$ when λ_2, λ_4 are finite. The chapter gives the second factorial moment of the number of crossings as an integral of a correlation term $A_u(s,t)$, together with its exact small-distance behaviour. For an isotropic Gaussian field the Hessian at a point is, up to scale, of the form $G - Z\,\mathrm{Id}_d$, where G is a GOE matrix and Z an independent Gaussian variable; the conditional law given the value of the field is given explicitly. These formulas feed directly into the study of critical points.

Chapter 4

Hermite expansions of level functionals

Summary

This chapter gives a simple and elementary presentation of the Hermite method. It avoids on purpose the heavy tools, such as multiple Wiener-Itô integrals and Malliavin calculus. The base is Wick's formula, which writes the expectation of a product of jointly Gaussian variables as a sum over pairings (diagrams). Hermite polynomials form an orthogonal basis for the Gaussian measure, and any square-integrable function of a Gaussian vector has a Hermite expansion.

The chapter builds Gaussian Hilbert spaces and their decomposition into orthogonal chaos. The Wick product and the diagram formula give a graphic way to compute moments by counting covariance pairings. With these tools the chapter gives an elementary proof of the fourth moment theorem inside a fixed chaos. It then states Arcones' inequality, which bounds the correlation of two Hermite functionals by a power of the correlation of their inputs. The chapter ends with a central limit theorem for the measure of the level set of a stationary Gaussian field, through the Hermite expansion of this measure.

Main results

Wick's formula states that, for centred jointly Gaussian variables $\xi_1, \dots, \xi_m$,

$$\mathbb{E}(\xi_1 \cdots \xi_m) = \sum_P \prod_{\{i,j\} \in P} \mathbb{E}(\xi_i \xi_j),$$

where the sum is over all pairings P of $\{1, \dots, m\}$ (0 if m is odd). The fourth moment theorem says that, for a sequence in a fixed chaos q, convergence in law to a standard Gaussian is equivalent to the convergence of the second moment to 1 and of the fourth moment to 3. Arcones' inequality states that, for g of Hermite rank τ and inputs with correlation bounded by ψ in absolute value,

$$\left|\mathbb{E}\big(g(\boldsymbol{\xi})\, g(\boldsymbol{\eta})\big)\right| \;\leq\; \mathbb{E}\big(g(\boldsymbol{\xi})^2\big)\, \psi^{\tau}.$$

Putting these together, the chapter proves that, under a Geman condition, a Kac-type condition and a mixing condition, the centred and normalised measure of the level set of a stationary Gaussian field converges in law to a centred Gaussian with finite variance as the domain grows.

Chapter 5

Critical points of isotropic Gaussian fields

Summary

This chapter counts and studies the critical points of an isotropic Gaussian field, on the sphere and in flat space. A critical point is a point where the gradient is zero; its index is the number of negative eigenvalues of the Hessian.

For fields on the sphere the chapter gives exact formulas for the mean number of critical points of a given index k below a level u. The formulas use the eigenvalues of a GOE matrix and the spectral moments of the field; in dimension two and three they reduce to closed forms. When the dimension grows the chapter studies the logarithmic asymptotics. There is a layered picture: at low levels one sees only minima, at higher levels minima and saddles, and so on. This is why optimisation in high dimension can be hard. The proof uses a large deviation principle for the eigenvalues of a GOE matrix; the result of Auffinger et al. (2013) is extended to the flat case.

The chapter then studies attraction and repulsion between critical points, through the second factorial moment. Different types of critical points behave differently: there is repulsion between maxima, a neutral behaviour between different types in dimension two, and attraction in higher dimension. These results extend the random plane wave case to general isotropic fields. The chapter ends with a multivariate central limit theorem for the number of critical points of each index, which applies in particular to the Euler characteristic.

Main results

The mean number of critical points of index k below a level u, on the sphere or in flat space, is given by an exact expression that is an expectation over the $(k+1)$-th eigenvalue of a GOE matrix, weighted by a Gaussian factor and a determinant. When the dimension d grows, the normalised logarithm of this mean number converges,

$$\lim_{d\to\infty} \frac{1}{d} \log \mathbb{E}\big(\mathrm{Crt}^k_{u\downarrow}\big) = \Theta^\star_k(v), \qquad v = \frac{u}{\sqrt{d+1}},$$

where $\Theta^\star_k$ is an explicit, piecewise rate function. For the correlation between two critical points at small distance ρ, the chapter gives the exact leading term, of order ρ^{2-d}, whose sign and size describe attraction or repulsion. Finally, as the domain grows, the vector of the numbers of critical points of each index converges in law to a multivariate Gaussian with finite, explicit limit covariance, under Geman and Arcones conditions on the field.

Chapter 6

Second maximum and sparse models

Summary

This chapter shows how the Kac-Rice method solves problems on the distribution of the maximum of a random field, with an application to high-dimensional statistics and sparse detection. The model is a Gaussian random field observed on a domain, and the question is to test whether its mean is zero against an unspecified, sparse alternative.

The chapter first recalls Reproducing Kernel Hilbert Spaces (RKHS) and the Mercer and Karhunen-Loève expansions. It uses them to define a class of sparse alternatives. The test is built from two quantities: the maximum λ_1 of the field, and the *second maximum* λ_2, which is the maximum of a normalised remainder after one regresses out the value and the gradient at the argument of the first maximum. The spacing between λ_1 and λ_2 tells whether a strong signal is present.

To get the test, the chapter proves an ad-hoc Kac-Rice formula that gives the exact law of λ_1 conditional on λ_2 and on the residual part Ω of the Hessian. From this law the *spacing test* is defined. When the variance is unknown, a Karhunen-Loève estimator of the variance is built, and the test becomes the *t-spacing test*, which uses a Student density instead of the Gaussian one. The methods are applied to spiked tensor PCA, to super-resolution deconvolution, and to the loss surface of two-layer neural networks.

Main results

Let ϕ be the standard Gaussian density and, for a symmetric matrix Ω, set $G_\Omega(\ell) = \int_\ell^{+\infty} \det(u\,\mathrm{Id} - \Omega)\,\phi(u)\,\mathrm{d}u$. The conditional law of λ_1 given (λ_2, Ω) has density proportional to $\det(\ell\,\mathrm{Id} - \Omega)\,\phi(\ell)$ on $\{\ell \geq \lambda_2\}$. The exactness of the spacing test follows: under the null hypothesis,

$$\mathcal{S}(\lambda_1, \lambda_2, \Omega) = \frac{G_\Omega(\lambda_1)}{G_\Omega(\lambda_2)} \sim \mathcal{U}(0,1),$$

so this ratio is an exact p-value. When the variance is unknown, the Karhunen-Loève estimator $\hat{\sigma}^2$ is, under the null, a scaled χ^2 variable, independent of the value and gradient of the field; replacing the Gaussian density by the Student density gives the t-spacing test, whose statistic is again uniform on $(0,1)$ under the null.

Chapter 7

Shot noise processes

Summary

This chapter treats a common non-Gaussian model, the shot noise process. A shot noise is a sum of kernel functions placed at random times (the points of a Poisson process) with random amplitudes. It models the addition of many random physical impulses.

The chapter proves three things. First, the process has a bounded marginal density, under conditions on the amplitude law and the kernel. Second, the moments of the supremum of the process and of its derivatives are uniformly bounded, under an integrability condition on the kernel and its derivatives. These two facts let one apply the general results of the earlier chapters. Third, the chapter proves central limit theorems: for the number of crossings of a level in dimension one, and for the measure of the level set of a shot noise field in higher dimension, both after centring and normalising, as the domain grows.

Main results

Under conditions on the amplitude and the kernel, the value $X(0)$ of the shot noise has a bounded density. Under an integrability condition on the kernel and its derivatives, together with a moment condition on the amplitude, the moments $\mathbb{E}\big(\|X^{(k)}\|_\infty^m\big)$ are finite. As a consequence, when the second moment of the number of crossings is finite, the centred and normalised number of crossings converges in law to a standard Gaussian,

$$\frac{N(t) - \mathbb{E}\, N(t)}{\sqrt{\operatorname{Var} N(t)}} \implies \mathcal{N}(0,1),$$

as $t \to \infty$. The same kind of central limit theorem holds for the centred and normalised $(d-1)$-measure of the level set of a shot noise field, as the domain grows in every direction.

Chapter 8

Related works

Summary

The last chapter is a review of topics that use the Kac-Rice formula and the Hermite method but are not treated in full in the book. The goal is to give the reader pointers to the literature.

The first topic is the winding number of a planar Gaussian process, that is the number of turns it makes around the origin. It is written with up-crossings and down-crossings, and a central limit theorem follows from the Kac-Rice formula and the Hermite method. The chapter then reviews random polynomials, a large field: random algebraic (Kac) polynomials, whose mean number of real roots behaves like $\frac{2}{\pi}\log d$, and random trigonometric polynomials, with the Gaussian and the non-Gaussian case, and with independent or dependent coefficients. The mean and the variance of the number of roots depend on the law and on the correlation of the coefficients.

Next the chapter reviews the nodal sets of random waves: Berry's random wave model, arithmetic random waves on the torus, and random spherical harmonics. The variance of the length of the nodal set grows like $\log k$ in dimension two but faster in dimension three. The chapter also reviews systems of random polynomial equations, in particular the Kostlan-Shub-Smale model: in the square case the mean number of real roots is $d^{m/2}$; in the rectangular case the zero set is a smooth manifold and one measures its volume. The chapter ends with topological events, such as the number of connected components of an excursion set, which are not local integrals; here the Nazarov-Sodin constant and tools from percolation are used. A short section on testing the anisotropy of a Gaussian field is also given.

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